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Sereno, M ; Peacock, J A

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DOI: <https://doi.org/10.1111/j.1365-2966.2006.10703.x>

Posted at the Zurich Open Repository and Archive, University of Zurich

ZORA URL: <https://doi.org/10.5167/uzh-154147>

Journal Article

Published Version

Originally published at:

Sereno, M; Peacock, J A (2006). Imprints of deviations from the gravitational inverse-square law on the power spectrum of mass fluctuations. *Monthly Notices of the Royal Astronomical Society*, 371(2):719-726.

DOI: <https://doi.org/10.1111/j.1365-2966.2006.10703.x>

# Imprints of deviations from the gravitational inverse-square law on the power spectrum of mass fluctuations

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Accepted 2006 June 13. Received 2006 May 19; in original form 2006 February 15

## ABSTRACT

Deviations from the gravitational inverse-square law would imprint scale-dependent features on the power spectrum of mass density fluctuations. We model such deviations as a Yukawa-like contribution to the gravitational potential and discuss the growth function in a mixed dark matter model with adiabatic initial conditions. Evolution of perturbations is considered in general non-flat cosmological models with a cosmological constant, and an analytical approximation for the growth function is provided. The coupling between baryons and cold dark matter across recombination is negligibly affected by modified gravity physics if the proper cut-off length of the long-range Yukawa-like force is  $\gtrsim 10 h^{-1}$  Mpc. Enhancement of gravity affects the subsequent evolution, boosting large-scale power in a way that resembles the effect of a lower matter density. This phenomenon is almost perfectly degenerate in power-spectrum shape with the effect of a background of massive neutrinos. Back-reaction on density growth from a modified cosmic expansion rate should, however, also affect the normalization of the power spectrum, with a shape distortion similar to the case of a non-modified background.

**Key words:** gravitation – cosmology: theory – dark matter – large-scale structure of Universe.

## 1 INTRODUCTION

General relativity has passed many important tests up to the length- and time-scales of the observable Universe (Peebles 2002, 2004). The body of evidence is quite impressive, considering the enormous extrapolation from the empirical basis, but the gravitational inverse-square law and its relativistic generalization are supported by high-precision tests from measurements in the laboratory, the Solar system and the binary pulsar only up to scales of  $\lesssim 10^{13}$  cm (Aldelberger, Heckel & Nelson 2003).

Despite the overall success of general relativity, non-standard theories have received much recent attention, largely motivated by finding alternative explanations for the ‘dark’ sector of the Universe. On one hand, MOND proposals make gravity stronger on scales of galaxies to explain flat rotation curves without dark matter (Milgrom 1983; Sanders 1998), on the other hand, alternative theories aim to find a mechanism for cosmic acceleration without dark energy, as a result of gravity leaking into higher dimensions on scales comparable to the horizon (Dvali, Gabadadze & Porrati 2000). Such radical modifications of gravitational physics may seem difficult to test, but our ability to make precise observations of cosmological dynamics on scales of  $\gtrsim 10 h^{-1}$  Mpc is now good enough to permit detailed tests of gravity theories even on these scales.

Examples of such work include White & Kochanek (2001), who determined constraints on the long-range properties of gravity from weak gravitational lensing cosmic shear. Constraints on Newton’s constant from the primordial abundances of light elements were discussed by Umezu, Kiyotomo & Masanobu (2005). The implications of MOND on large-scale structure in a Friedmann–Lemaître–Robertson–Walker (FLRW) universe were discussed in Nusser (2002). Recently, proposals have been made to extend these theories to a general covariant framework (Bekenstein 2004) and evolution of perturbations in this theory has been considered (Skordis et al. 2005). Finally, large galaxy surveys have been analysed looking for signatures in the power spectrum due to deviations from the inverse-square law (Sealfon, Verde & Jimenez 2005; Shirata et al. 2005). In the present paper, we will concentrate on this last aspect.

We follow a number of recent authors, who have considered additional contributions to the gravitational potential in the form of Yukawa-like terms (White & Kochanek 2001; Peebles 2002; Sealfon et al. 2005; Shirata et al. 2005). We consider the growth function and the power spectrum of matter density fluctuations in these Yukawa-like models; we extend previous work by removing a restriction to fixed cosmological parameters and also explore some possible back-reactions from a modified cosmic expansion rate. We still adopt the cold dark matter (CDM) model for structure formation with adiabatic initial conditions, but we also consider variants due to hot dark matter. The paper is structured as follows. In

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Section 2, we introduce Yukawa-like contributions to the gravitational potential and study the linear perturbation equation and the growth function. In Section 3, we give some insights on the coupled growth of baryons and CDM. In Section 4, we illustrate the effect of modified gravity on the power spectrum, whereas Section 5 is devoted to the interplay between mixed dark matter (MDM) and gravity physics. Growth of perturbations in a model with modified cosmic expansion is addressed in Section 6. Section 7 contains some final considerations.

## 2 LINEAR PERTURBATION EQUATION

The dynamics of linear perturbations can be easily extracted from a Newtonian approach by writing down the fundamental equations of fluid motion (e.g. Peacock 1999, chapter 15). Usually, acceleration is related to mass density via Poisson's equation, but here we allow for deviations. We model the weak field limit of the gravitational potential,  $\phi$ , as a sum of a Newtonian potential and a Yukawa-like contribution (see White & Kochanek 2001, and references therein):

$$\phi = \alpha_{\text{Nwt}}\phi_{\text{Nwt}} + \alpha_{\text{Ykw}}\phi_{\text{Ykw}}. \quad (1)$$

The factors  $\alpha_{\text{Nwt}}$  and  $\alpha_{\text{Ykw}}$  account for deviations of the effective gravitational coupling constant from  $G_{\text{N}}$ , the value actually observed on small scales.  $\phi_{\text{Nwt}}$  is the solution of Poisson's equation, whereas  $\phi_{\text{Ykw}}$ , which describes forces mediated by massive particles, is the solution of the modified Poisson equation; in comoving coordinates, this can be written as

$$\frac{\nabla^2 \phi_{\text{Ykw}}}{a^2} - \frac{\phi_{\text{Ykw}}}{\lambda^2} = 4\pi G_{\text{N}} \rho_{\text{M}} \delta(x, t), \quad (2)$$

with  $a$  the cosmic scalefactor,  $\rho_{\text{M}}$  the total matter density,  $\delta$  the mass density contrast and  $\lambda$  a proper length cut-off. The potential in equation (1) is  $\propto 1/x$  both on small scales ( $x \ll \lambda/a$ ), with an effective coupling constant  $G = (\alpha_{\text{Nwt}} + \alpha_{\text{Ykw}})G_{\text{N}}$ , and on large scales, with  $G_{\infty} = \alpha_{\text{Nwt}}G_{\text{N}}$ . Putting

$$\alpha_{\text{Nwt}} \equiv 1 + \Delta\alpha \quad (3)$$

and  $\alpha_{\text{Ykw}} = -\Delta\alpha$ ,  $\phi$  is in numerical agreement with results on the scale of the Solar system, whereas  $G_{\infty} = (1 + \Delta\alpha)G_{\text{N}}$ . For  $\Delta\alpha > 0$  ( $< 0$ ), gravity is enhanced (suppressed) on large scales. The potential in equation (1) can be derived in a relativistic gravity model that obeys the equivalence principle (Zhytnikov & Nester 1994). Although the Yukawa-like form is quite specific, all long-range deviations can be characterized empirically by an amplitude,  $\Delta\alpha$ , and a length-scale,  $\lambda$ .

The basic equation for the linear evolution of the Fourier component  $\delta_k$  of the density fluctuation, with comoving wavenumber  $k$ , is

$$\ddot{\delta}_k + 2H(t)\dot{\delta}_k = -\frac{k^2\phi_k}{a^2}, \quad (4)$$

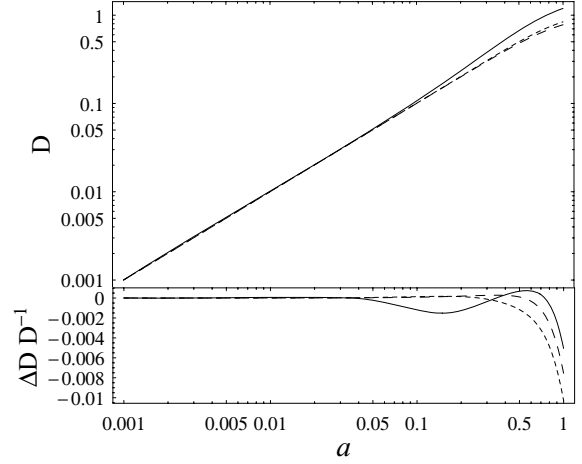
where dot denotes time derivative and  $H \equiv \dot{a}/a$  is the time-dependent Hubble term. In Fourier space, the potential in equation (1) is (White & Kochanek 2001; Sealfon et al. 2005)

$$\phi_k = -\frac{4\pi G_{\text{N}} a^2 \rho_{\text{M}} \delta_k}{k^2} \mathcal{Y}(a/a_{\lambda}; \alpha_{\text{Nwt}}, \alpha_{\text{Ykw}}), \quad (5)$$

with

$$\mathcal{Y} \equiv \alpha_{\text{Nwt}} + \frac{\alpha_{\text{Ykw}}}{1 + (a/a_{\lambda})^2} \quad (6)$$

$$= 1 + \Delta\alpha \frac{(a/a_{\lambda})^2}{1 + (a/a_{\lambda})^2} \quad (7)$$



**Figure 1.** Linear growth rate as a function of the scalefactor  $a$  in a flat  $\Lambda$ CDM model with  $\Omega_{\text{M}0} = 0.25$  for Yukawa-model parameters  $\Delta\alpha = 0.5$  and  $\lambda = 10$  Mpc. Extra growth is effective at late times and on large scales. Upper panel: dependence on scale. Extra growth is activated for  $a \gtrsim k\lambda$ . The full, short-dashed and dashed lines are for  $k = 10^{-2}$ ,  $5 \times 10^{-2}$  and  $10^{-1}$  Mpc $^{-1}$ , respectively. Lower panel: fractional residuals between the above numerical results and the fitting formula.

and  $a_{\lambda} \equiv \lambda k$ . From equations (4) and (5), we obtain (Sealfon et al. 2005; Shirata et al. 2005)

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \frac{3}{2} \frac{\Omega_{\text{M}0} H_0^2}{a^3} \mathcal{Y}(a/a_{\lambda}; \alpha_{\text{Nwt}}, \alpha_{\text{Ykw}}) \delta_k = 0, \quad (8)$$

where  $\Omega_{\text{M}0}$  is the present density of total matter in units of the critical one and  $H_0$  is the Hubble constant, parametrized in the following as  $100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ . A deviation from the inverse-square law thus causes the rate of growth of the Fourier amplitudes to depend on wavelength. The inclusion of a Yukawa-like contribution imprints the scale  $\lambda/a_0$  on the power spectrum. Let us examine the growth between an initial epoch,  $a_{\text{in}}$ , and the actual time,  $a_0$ , in more detail. On a very large scale, i.e.  $k \ll a_{\text{in}}/\lambda$  and  $\mathcal{Y} \sim 1 + \Delta\alpha$ , modes always experience the modified growth due to the effective gravitational constant. Intermediate scales, initially below the maximal comoving cut-off length  $\lambda/a_{\text{in}}$ , will cross the scale for modified growth at some time  $a_{\text{in}} \lesssim a_{\lambda} \lesssim a_0$ . Finally, very small scales  $k \gg a_0/\lambda$  will evolve in a Newtonian potential. As a result, the growth of fluctuations is no longer independent of scale, even at low redshift. Some examples of the growth function are plotted in Fig. 1.

Equation (4), with  $\phi$  given as in equation (5), has been solved analytically for an Einstein-de Sitter (EdS) model ( $\Omega_{\text{M}0} = 1$ ,  $\Omega_{\text{K}0} = 0$ ) in terms of hypergeometric functions in Shirata et al. (2005).<sup>1</sup> Here, we want to examine approximate solutions in the general case with cosmological constant. Let us begin with the EdS model. A Yukawa-like contribution with  $\Delta\alpha > 0$  ( $< 0$ ) enhances (suppresses) the growth of fluctuations on scales sufficiently larger than  $\lambda$ . The growth law becomes

$$\delta \propto a^{1+p_{\Delta\alpha}}, \quad p_{\Delta\alpha} = \frac{\sqrt{25 + 24\Delta\alpha} - 5}{4}. \quad (9)$$

The growth function,  $D$ , of the CDM+baryons density fluctuations can be approximated by smoothly interpolating between the analytic

<sup>1</sup> We refer to Section 6 for considerations about modified cosmic expansion rate and adopt here a standard FLRW framework.

results. We propose, for the growing mode,

$$D_{\text{Ykw}}^{\text{EdS}}(a; \Delta\alpha, \lambda) = \delta_{\text{in}} \frac{a}{a_{\text{in}}} \left[ 1 + \left( \frac{a/a_{\text{in}}}{1 + c_2 a_\lambda/a_{\text{in}}} \right)^{c_1} \right]^{p_{\Delta\alpha/c_1}}, \quad (10)$$

where the normalization has been chosen to be  $D = \delta_{\text{in}} a/a_{\text{in}}$  at early times  $a \sim a_{\text{in}}$  and the coefficients  $c_1$  and  $c_2$  represent a fit to numerical evolution. For  $a_{\text{in}} = 10^{-3}$ , we get

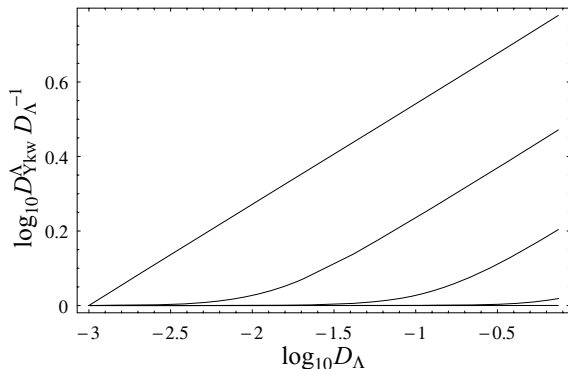
$$c_1 = 1.94088 \quad (11)$$

$$c_2 = 1.42350 - 0.170668\Delta\alpha. \quad (12)$$

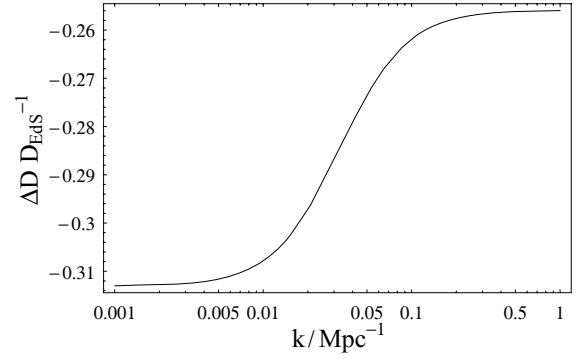
The fitting formula works quite well, for  $10^{-2} \lesssim k/(h \text{ Mpc}^{-1}) \lesssim 1$ , for the parameter range  $-0.25 \lesssim \Delta\alpha \leq 1$  and  $2 \lesssim \lambda/(h^{-1} \text{ Mpc}) \lesssim 100$ , with fractional residuals always under 1 per cent.

The growth function changes at late times when  $\Omega_{\text{M}0} \neq 1$  since, after matter ceases to dominate the expansion rate, fluctuation growth halts. This must be considered together with the late-time effect due to deviations from the inverse-square law since the comoving scale for modified growth,  $\lambda/a$ , moves to smaller values with increasing time. It is easy to account for both of these effects. Let  $D_\Lambda$  denote the growth function for a model with non-zero curvature and cosmological constant but in the absence of Yukawa-like terms. Then, by the replacement of  $a/a_{\text{in}}$  with  $D_\Lambda$  in equation (10), we can approximate all the dependence of the scale-dependent growth function on time, curvature and cosmological constant in the presence of a modified gravitational potential. A useful approximation for  $D_\Lambda$  can be found in Carroll, Press & Turner (1992). In the lower panel of Fig. 1, we compare the numerical evaluation to the fitting formula. The accuracy is quite high, with performance better than 1 per cent for a parameter space similar to the EdS case. The fitting formula approximates the asymptotic limit on very large scales or late times, i.e.  $a \gtrsim a_\lambda$ , as  $D_\Lambda^{1+p_{\Delta\alpha}}$ . This is a natural generalization of the  $a^{1+p_{\Delta\alpha}}$  form that applies for the EdS model. Although the result is no longer exact if  $\Lambda_{00} \neq 0$ , it is very accurate in practice. In Fig. 2, we can see how the linear growth index changes from the usual Newtonian trend to an enhanced index for decreasing wave numbers and decreasing redshifts and how well the  $D_\Lambda^{1+p_{\Delta\alpha}}$  approximation works at late times.

In the presence of a Yukawa-like term, the suppression due to a cosmological constant with respect to the EdS model is most effective at large scales, in contrast to the Newtonian case, where



**Figure 2.** Linear growth rate, normalized to the pure Newtonian case  $D_\Lambda(\Delta\alpha = 0)$ , as a function of  $D_\Lambda$ . The background cosmology is a flat  $\Lambda$ CDM model with  $\Omega_{\text{M}0} = 0.25$ , whereas the Yukawa-model parameters are fixed to  $\Delta\alpha = 0.5$  (i.e.  $p_{\Delta\alpha} \sim 0.27$ ) and  $\lambda = 10 \text{ Mpc}$ . The lines from above to below correspond to  $k = 10^{-6}, 10^{-3}, 10^{-2}, 10^{-1}$  and  $1 \text{ Mpc}^{-1}$ , respectively. The growth function evolves as  $D_\Lambda$  at very early times or small scales, and as  $D_\Lambda^{1+p_{\Delta\alpha}}$  when  $a \gtrsim \lambda k$ .



**Figure 3.** Fractional deviation of the growth function, at the present time  $a = 1$ , between an EdS model and a flat  $\Lambda$ CDM model with  $\Omega_{\text{M}0} = 0.25$ . The Yukawa-model parameters are  $\Delta\alpha = 0.5$  and  $\lambda = 20 \text{ Mpc}$ . The suppression due to a cosmological constant is scale dependent.

the suppression is scale-independent. Let us consider the case  $\Delta\alpha > 0$ . Whereas a Yukawa-like contribution is effective at late times and on large scales, extra growth is inhibited by a cosmological constant when matter becomes subdominant in the dynamics. In Fig. 3, we plot the fractional deviation in the growth function between an EdS cosmology and a flat  $\Lambda$ CDM model.

The hypotheses used above are standard in modelling the effect of a Yukawa-like contribution to the gravitational potential on the growth function (Sealfon et al. 2005; Shirata et al. 2005). In the next sections, we will relax some assumptions and extend the analysis to MDM models.

### 3 BARYONS

Baryons imprint characteristic features into the power spectrum as the direct result of small density fluctuations in the early Universe prior to recombination, when they are tightly coupled with the photons and share in the same pressure-induced oscillations (Eisenstein & Hu 1998). As far as the Yukawa-like potential affects all kinds of matter in the same way, the modified coupled equations for perturbation growth in a mixture of baryons and CDM can be written as

$$\begin{pmatrix} \delta_B'' \\ \delta_C'' \end{pmatrix} + 2H \begin{pmatrix} \delta_B' \\ \delta_C' \end{pmatrix} = 4\pi G \frac{\rho_{\text{M}}}{\Omega_{\text{M}0}} \mathcal{Y} \begin{bmatrix} \Omega_{\text{B}0} - k^2/(k_J^2 \mathcal{Y}) & \Omega_{\text{C}0} \\ \Omega_{\text{B}0} & \Omega_{\text{C}0} \end{bmatrix} \begin{pmatrix} \delta_B \\ \delta_C \end{pmatrix}, \quad (13)$$

where  $\delta_B$  and  $\delta_C$  are the density fluctuations in baryons and CDM, respectively,  $k_J \equiv a\sqrt{4\pi G_N \rho_{\text{M}}}/c_s$  is the comoving Jeans wave number in a Newtonian potential and  $c_s$  is the sound speed. On a small scale, baryon pressure causes the growth rates in baryons and CDM to differ. Gravity-driven growth is not allowed for baryons above a critical wavenumber, if  $k/(k_J\sqrt{\mathcal{Y}}) \gg 1$ . This condition can be re-written for small values of  $\Delta\alpha$  as

$$k \gg k_J \left[ 1 + \frac{\Delta\alpha/2}{1 + (k_J\lambda/a)^2} \right] + \mathcal{O}(\Delta\alpha^2). \quad (14)$$

A positive value of  $\Delta\alpha$  pushes the domain of predominance of pressure over gravity to smaller lengths. When pressure overcomes gravity, a Yukawa-like term does not affect the dynamics of baryons. As usual, their main behaviour will be oscillatory, with slowly declining sound waves described by the Wentzel–Kramers–Brillouin (WKB) solution (e.g. Peacock 1999, equation 15.60).

Either on large scales or after the Compton drag epoch, when the sound speed drops by a large factor, pressure effects are negligible. Solutions of equation (13) are simple since the matrix in the driving term has time-independent eigenvectors:  $(1, 1)$  and  $(\Omega_{C0}, -\Omega_{B0})$ . Hence, a coupled perturbation will be quickly dominated by the fastest-growing mode with  $\delta_B = \delta_C$  and baryons quickly catch up with the CDM.

Since we have been considering a proper length  $\lambda \gtrsim 10 h^{-1}$  Mpc, the interaction of matter and radiation at decoupling is not affected by modified gravity: the cut-off at this time is  $\gtrsim 10 h^{-1}$  Gpc in co-moving units, so its effects appear on a scale too large to be observed. Prior to the drag epoch, the comoving baryonic Jeans scale,  $\lambda_J \equiv 2\pi/k_J$ , tracks the sound horizon, which is of the order of  $100 h^{-1}$  Mpc at  $a \sim 10^{-3}$ . Hence, for a proper cut-off length  $\lambda \gtrsim 10 h^{-1}$  Mpc, the effects of a Yukawa-like term on the effective Jeans wavenumber, equation (14), are really negligible. After the drag epoch, the oscillatory mode of baryons matches on to a mixture of pressure-free growing and decaying modes. Such modes are given by the solutions of the perturbation equation discussed in Section 2 for a  $\Omega_{M0} = 1$  matter-dominated universe. Because the form of the WKB solution is preserved and because the modes after recombination are affected by modified gravity only at later times, velocity overshoot is still produced with the output after the drag epoch being maximized when the wave consists of a pure velocity perturbation.

Although the position of the acoustic peaks is nearly unaffected, later evolution can still affect the scales of baryonic oscillations for small values of  $\lambda$  and large values of  $\Delta\alpha$ . A Yukawa-like term can alter the height of peaks, with the first peak enhanced with respect to the second one when  $\Delta\alpha > 0$ . Baryonic acoustic oscillations have been proposed as probes of dark energy (Blake & Glazebrook 2003; Seo & Eisenstein 2003; Matsubara 2004), but most of the attention is on position rather than height, so that the effect of modified gravity is negligible at the present status of accuracy of galaxy redshift surveys (Cole et al. 2006).

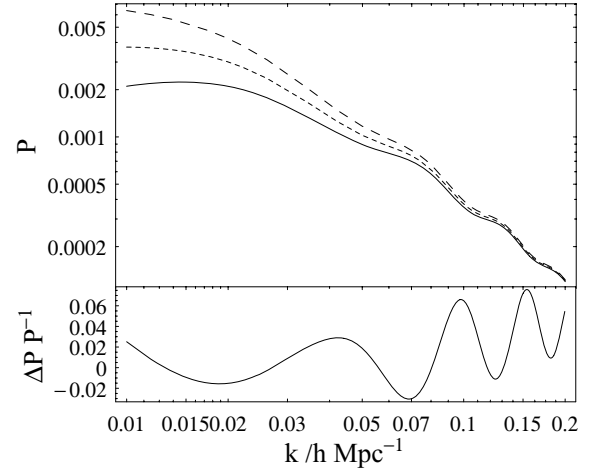
#### 4 POWER SPECTRUM

The linear power spectrum can be constructed from the transfer function as

$$P(k, a) \propto k^n T^2(k) \left[ \frac{D(z, a)}{D(z, a_{in})} \right]^2, \quad (15)$$

where  $T(k)$  is the matter transfer function at the drag epoch, and  $n$  is the initial power-spectrum index, equal to 1 for a scale invariant spectrum. As discussed in the previous section, we can assume that standard theory provides the transfer function at the last scattering surface. In what follows, we will use the fitting formulae for  $T(k)$  in Eisenstein & Hu (1998, 1999).

A positive long-range contribution to the gravitational potential enhances clustering on scales larger than  $\lambda$ . The shape distortion of the power spectrum can be quite significant (see Fig. 4). A Yukawa-like contribution with  $\Delta\alpha > 0$  changes the spectrum in a way that resembles a lower matter density, with a near degeneracy between the pair  $\{\Delta\alpha, \lambda\}$  and  $\Omega_{M0}$ . An illustrative example is contained in the lower panel of Fig. 4. Larger fractional residuals appear at the scale of the acoustic peaks, since in our example we have varied  $\Omega_{M0}$  but kept  $\Omega_{B0}$  fixed. As a matter of fact, the location of baryonic oscillations is nearly independent of  $\Delta\alpha$  and could break the compensation. As well as a change in density parameters, a Yukawa-like contribution can mimic a variation in the primaeval spectral index, with values of  $n < 1$  nearly degenerate with  $\Delta\alpha > 0$ , over a large range of wave numbers.



**Figure 4.** Effect of a Yukawa-like term on the power spectrum. Upper panel: the full, dashed and long-dashed lines are for  $\Delta\alpha = 0, 0.3$  and  $0.6$ , respectively, with  $\lambda$  being fixed at  $10 h^{-1}$  Mpc in a flat  $\Lambda$ CDM model with  $h = 0.7$ ,  $\Omega_{M0} = 0.25$ ,  $\Omega_{B0} = 0.05$  and  $n = 1$ . Units of the power spectrum are arbitrary. An enhancement of gravity ( $\Delta\alpha > 0$ ) boosts large-scale power. Lower panel: fractional residuals between the pure Newtonian case of the upper panel and a re-normalized power spectrum in a flat model with  $h = 0.7$ ,  $\Omega_{M0} = 0.30$ ,  $\Omega_{B0} = 0.05$ ,  $n = 1$  and Yukawa-model parameters  $\Delta\alpha = 0.19$ ,  $\lambda = 5 h^{-1}$  Mpc. The effect of enhanced gravity is similar to that of lower matter density.

The degeneracy between matter density and Yukawa parameters could be quite significant when determining cosmological parameters, since power-spectrum analyses provide one of the most important direct constraints on the value of  $\Omega_{M0}$ , whereas other methods that are nearly independent of a Yukawa-like contribution, i.e. methods based on the cosmic microwave background radiation or supernovae of Type Ia, are mostly sensitive to other combinations of cosmological parameters.

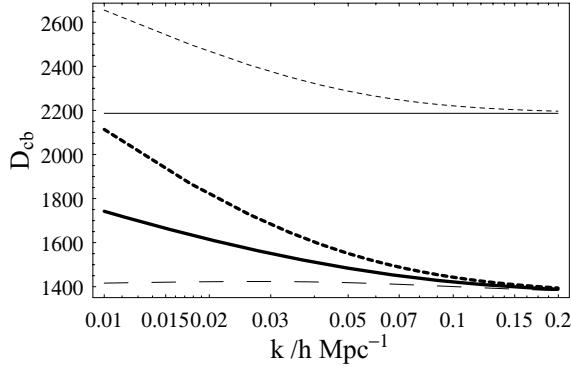
#### 5 MIXED DARK MATTER

The introduction of massive neutrinos or other forms of hot dark matter couples the spatial and temporal behaviour of perturbation growth (Ma 1996; Hu & Eisenstein 1998; Eisenstein & Hu 1999). On scales smaller than the free-streaming length, neutrinos do not trace CDM and baryon perturbations, which in turn grow more slowly because of the reduction of the gravitational source. As neutrinos slow down and their free-streaming scale shrinks to even smaller scales with increasing time, the transfer function acquires a non-trivial time dependence.

As has been discussed in Hu & Eisenstein (1998) and Eisenstein & Hu (1999), the transfer function in a MDM cosmology with CDM, baryons and massive neutrinos can be decomposed into a scale-dependent growth function that incorporates all post-recombination effects, and a time-independent function that represents the perturbations around recombination. The late-time evolution of density-weighted CDM and baryons perturbations can be accurately fitted by (Hu & Eisenstein 1998)

$$D_{cb}(y, k) = \left\{ 1 + \left[ \frac{D_{\Lambda}(y)}{1 + y_{fs}(k)} \right]^{0.7} \right\}^{p_{cb}/0.7} D_{\Lambda}(y)^{1-p_{cb}}, \quad (16)$$

where  $D_{\Lambda}$  is the growth function in the absence of neutrino free-streaming,  $y \equiv a/a_{eq}$ , with  $a_{eq}$  the epoch of matter-radiation equality,  $y_{fs}$  is the free-streaming epoch as a function of scale and  $p_{cb} \equiv$



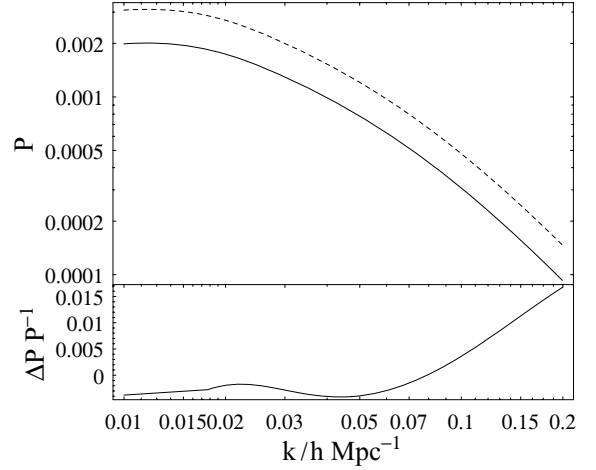
**Figure 5.** Scale-dependent growth function in a MDM cosmology, evaluated at the present epoch. We consider a flat  $\Lambda$ CDM model with  $h = 0.7$ ,  $\Omega_{M0} = 0.25$ . Full lines represent Newtonian cases ( $\Delta\alpha = 0$ ); thick lines include neutrinos (three species and  $f_\nu = 0.10$ ); short-dashed lines account for a Yukawa-like term with  $\Delta\alpha = 0.20$  and  $\lambda = 12 h^{-1} \text{ Mpc}$ . Scale-dependent features due to massive neutrinos are nearly degenerate with modified gravity. The long-dashed curve stands for the growth function with  $\Delta\alpha = -0.20$ ,  $\lambda = 12 h^{-1} \text{ Mpc}$ , three neutrino species and  $f_\nu = 0.05$ . Effects due to neutrinos can be cancelled by a Yukawa-like term with  $\Delta\alpha < 0$ .

$(5 - \sqrt{1 + 24f_{cb}})/4$  with  $f_{cb}$  the matter fraction in CDM+baryons. The fitting formula in equation (16) interpolates across  $y_{fs}$  between the suppressed logarithmic growth rate at small scales, which is  $1 - p_{cb}$  in a EdS model, and the usual growth rate at sufficiently large scales. An analogous formula for the growth rate for the CDM+baryon+neutrino perturbations can be found in Hu & Eisenstein (1998). If we replace  $D_\Lambda$  with the growth function  $D_{Ykw}^\Lambda$ , discussed in Section 2,  $D_{cb}$  will contain all the dependence of the transfer function on time, curvature, cosmological constant and Yukawa-like contribution in the presence of neutrino free-streaming.<sup>2</sup> An amplitude  $\Delta\alpha > 0$  has a similar effect to that of a non-zero neutrino mass, both reinforcing large-scale evolution. Some examples of growth functions are plotted in Fig. 5. Scale-dependent features due to neutrinos can be almost perfectly cancelled by a Yukawa-like term.

It turns out that features in the power spectrum due to the transition between free-streaming and infall of massive neutrinos have much in common with imprints induced by enhanced gravity: see Fig. 6, where the fitting formulae from Eisenstein & Hu (1999), which do not account for baryonic oscillations, have been used. An enhancement of gravity ( $\Delta\alpha > 0$ ) can distort the shape of the power spectrum in nearly the same way as a non-zero neutrino mass, both boosting large-scale power. Even at low redshifts, some scales can be still either in the free-streaming regime or below the  $\lambda$ -cut-off, so there is an interplay between the two effects. A true deviation from the inverse-square law can be interpreted as a significant neutrino fraction  $f_\nu$  of the total matter density (see the lower panel of Fig. 6). In a low-density flat model with three nearly degenerate neutrino species and  $f_\nu \lesssim 0.10$ , an adequate approximation for such a degeneracy, is, if  $\lambda \lesssim 50 h^{-1} \text{ Mpc}$ ,

$$(f_\nu)_{\text{apparent}} \simeq \left[ 0.312 - 0.252 \left( \frac{\lambda}{50} \right) \right] (\Delta\alpha)_{\text{true}} \quad (17)$$

<sup>2</sup> Note that the growth functions in Hu & Eisenstein (1998) have been normalized to  $D = a/a_{eq}$  at early time, so that  $D_{Ykw}$  must be conveniently re-scaled.



**Figure 6.** The effect of massive neutrinos on the shape of the power spectrum is nearly degenerate to that of a Yukawa-like contribution. We consider a flat  $\Lambda$ CDM model with  $h = 0.7$ ,  $\Omega_{M0} = 0.25$ ,  $\Omega_{B0}/\Omega_{M0} = 0.15$  and  $n = 1$ . Upper panel: the full line represents the power spectrum of the density weighted CDM, baryon and neutrino perturbations for  $f_\nu = 0.05$ , three neutrino species and Newtonian potential ( $\Delta\alpha = 0$ ); the dashed line stands for  $\Delta\alpha = 0.19$  and  $\lambda = 10 h^{-1} \text{ Mpc}$  without neutrinos. The units for the power spectrum are arbitrary. Lower panel: fractional residuals after re-scaling.

with the proper length  $\lambda$  in units of  $h^{-1} \text{ Mpc}$ . For a smaller cut-off proper length, a larger deviation  $\Delta\alpha$  is needed to mimic the effect of massive neutrinos. This compensation works up to the scales where non-linear or redshift-space effects begin to affect the shape of the power spectrum, i.e. for  $k \lesssim 0.1 h \text{ Mpc}^{-1}$  (Meiksin, White & Peacock 1999).

The fractional contribution of neutrinos to the total mass density has been derived from the 2dF Galaxy Redshift Survey (2dFGRS) data (Elgarøy et al. 2002). Adding prior information from independent cosmological probes, Elgarøy et al. (2002) found  $f_\nu < 0.13$  (at 95 per cent confidence), i.e. an upper bound on the total neutrino mass  $m_\nu < 1.8 \text{ eV}$ . Spergel et al. (2003) combined 2dFGRS data with measurements from *Wilkinson Microwave Anisotropy Probe* (WMAP) and obtained a tight limit  $m_\nu < 0.69 \text{ eV}$ . These results are strongly dependent on the assumed priors (Elgarøy & Lahav 2003) and could be significantly affected by a long-range deviation from the inverse-square law. Since a long-range enhancement could equally lead to extra large-scale power, the upper bound on the neutrino mass would be over-(under) estimated if  $\Delta\alpha > 0$  ( $< 0$ ).

This degeneracy applies only at one epoch, and the evolution of  $P(k)$  will be different in the two cases. Although neutrinos mainly affect the growth function, they also contribute a residual suppression of power in the transfer function on small scales at the drag epoch. On the other hand, a Yukawa-like contribution, which is characterized by a proper length cut-off, is only effective at later times.

## 6 MODIFIED COSMIC EXPANSION

In the previous sections, we have followed the usual approach by assuming that a modified gravitational potential only affects fluctuations while leaving the background cosmology intact (White & Kochanek 2001; Nusser 2002; Sealfon et al. 2005; Shirata et al. 2005). We now want to address the effect of a perturbed cosmological expansion rate, due to modified gravity physics, on the dynamics of large-scale structure. In fact, Lagrangian-based

theories of modified Newtonian gravity, in which the FLRW background cosmology still holds in the absence of fluctuations, have been proposed (Sanders 2001), but such frameworks are still controversial. Possible cosmological implications of new long-range gravitational interactions have been discussed within the framework of Newtonian cosmology. D’Olivo & Ryan (1987) showed that for a homogeneous, isotropic universe, with a Yukawa-type term in the gravitational interaction, a Newtonian cosmology exists and the equations of motions are the same as for the ordinary Newtonian case, i.e. the Friedmann equations. The only difference is that the effective gravitational constant at large distance,  $G_\infty$ , enters the equations instead of the local measured value,  $G_N$ . Although no proof has been produced that such predictions from a Newtonian cosmology can be used in full relativistic cosmology, this modified FLRW model with an effective coupling constant provides a motivated framework where implications of a modified cosmic expansion rate on density fluctuations can be explored.

We should therefore consider using the standard form of Friedmann equations, with  $G_\infty$  in place of  $G_N$ , and the standard cosmological parameters, with the only change being to normalize energy densities to a re-scaled cosmological critical density,  $\rho_{cr}^{Ykw} \equiv 3H_0^2/(8\pi G_\infty)$ . Hence, the usual cosmological parameters are related to the new ones by

$$\Omega_{i0} = \frac{G_N}{G_\infty} \Omega_{i0}^{Ykw} = \frac{\Omega_{i0}^{Ykw}}{\alpha_{Nwt}}. \quad (18)$$

Within this model, local estimates of the matter density parameter, as those from cluster galaxy mass and luminosity, should be re-scaled when being compared to values determined from cosmic distances.

A Yukawa-type potential causes a change in the cosmic expansion rate of the background. The effect on linear perturbations is easily understood. Now, also the Hubble drag term in equation (4) depends on the effective coupling constant  $G_\infty$ , whereas the form of the driving term on the right-hand side is not affected. A positive long-range interaction ( $\Delta\alpha > 0$ ) enhances the drag term in equation (4), so that growth is suppressed with respect to the non-modified cosmological dynamics. We rewrite the linear perturbation equation as

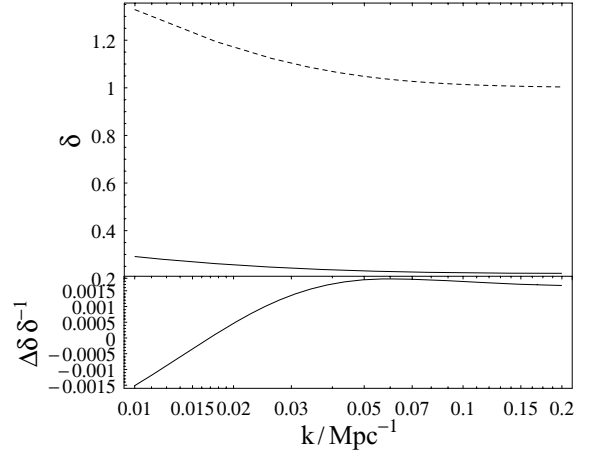
$$\ddot{\delta}_k + 2H\dot{\delta}_k + \frac{3}{2} \frac{\Omega_{M0}^{Ykw} H_0^2}{a^3} \frac{\mathcal{V}}{\alpha_{Nwt}} \delta_k = 0, \quad (19)$$

with

$$H(a) = H_0 \sqrt{\Omega_{M0}^{Ykw} a^3 + \Omega_{\Lambda 0}^{Ykw} + \Omega_{K0}^{Ykw} a^2}. \quad (20)$$

As can be seen after comparison with equation (8), equation (19) cannot be simply obtained by substituting each cosmological density parameter  $\Omega_{i0}$  with the corresponding  $\Omega_{i0}^{Ykw}$ , since the driving term is suppressed by a factor of  $1/\alpha_{Nwt}$  due to the new role played by the effective coupling constant in the background dynamics. Let us consider the equivalent of an EdS model, i.e. a flat model with  $\Omega_{M0}^{Ykw} = 1$ . The perturbation equation can now be solved in term of hypergeometric functions,

$$\begin{aligned} \delta_k = & C_1 \left( \frac{a}{a_\lambda} \right)^{-3/2+p_\beta} \\ & \times {}_2F_1 \left[ \frac{p_\beta}{2} - \frac{5}{4}, \frac{p_\beta}{2}, p_\beta - \frac{1}{4}, -\left( \frac{a}{a_\lambda} \right)^2 \right] \\ & + C_2 \left( \frac{a}{a_\lambda} \right)^{1-p_\beta} \\ & \times {}_2F_1 \left[ -\frac{p_\beta}{2}, \frac{5}{4} - \frac{p_\beta}{2}, \frac{9}{4} - p_\beta, -\left( \frac{a}{a_\lambda} \right)^2 \right], \end{aligned} \quad (21)$$



**Figure 7.** The growth function, evaluated at the present epoch, in a model with modified background expansion. The shape of scale-dependent features due to a Yukawa-term are negligibly affected by a modified cosmic rate. Upper panel: the full line stands for a modified EdS model with  $\Delta\alpha = 0.5$  and  $\lambda = 20$  Mpc; the short-dashed line represents the growth function in a non-modified EdS model with  $\Delta\alpha = 0.4$  and  $\lambda = 21$  Mpc. Lower panel: fractional residuals after re-scaling.

where  ${}_2F_1$  is the hypergeometric function,  $C_1$  and  $C_2$  are normalization constants determined by the initial conditions and  $p_\beta \equiv (5 - \sqrt{25 - 24\beta})/4$  with  $\beta \equiv \Delta\alpha/(1 + \Delta\alpha)$ . The second term in equation (21) is the growing mode which interpolates between  $\delta \propto a^{1-p_\beta}$  at small scales ( $a \ll a_\lambda$ ) and the usual  $\delta \propto a$  at late times and/or large scales. Due to the modified background, a Yukawa-like contribution with  $\Delta\alpha > 0$  ( $< 0$ ) determines suppression (extra-growth) at small scales, whereas there is no variation on large scales.

The global effect on the shape of growth function is actually similar to the case of a non-modified cosmic expansion rate discussed in the previous sections, the main difference being an overall normalization factor. The two cases are really indistinguishable when the differences in the growth index between large and small scale, i.e.  $p_{\Delta\alpha}$  and  $p_\beta$ , are equal. Let us further explore the near-degeneracy between modified and non-modified cosmic expansion by considering the EdS model in the two cases. We recall that the different definitions of critical density,  $\Omega_{M0}^{Ykw} = 1$  or  $\Omega_{M0} = 1$ , correspond to different values of  $\rho_{M0}$ . A value  $\Delta\alpha$  in a non-modified background distorts the power spectrum in nearly the same way as  $\Delta\alpha' \equiv (3\Delta\alpha - 4p_{\Delta\alpha}^2)/[4p_{\Delta\alpha}^2 + 3(1 - \Delta\alpha)]$  in a modified cosmology with approximately the same cut-off proper length. In Fig. 7, we show the degeneracy in the shape of the growth function between modified and non-modified cosmic expansion rate. Differences show themselves only in the normalization factor, with growth of fluctuation suppressed for enhanced long-range gravitational interaction, due to the boosted cosmic expansion.

## 7 CONCLUSIONS

We have discussed the dynamics of large-scale structure in the presence of a Yukawa-like contribution to the gravitational potential. The strength parameter and its cut-off length can introduce peculiar scale-dependent features in the shape of the power spectrum of density fluctuations. The main imprint of enhanced gravity is enhanced large-scale power, in a way that resembles the effect of a lower matter density. If the extra long-range force couples to all kinds of matter in the same way, then baryons and CDM evolve together

after recombination. Furthermore, for  $\lambda \gtrsim 10 h^{-1}$  Mpc, the transfer function at the Compton drag epoch is not significantly affected by Yukawa-like contributions.

With regard to evolution, enhanced gravity acts very similarly to hot dark matter: both effects yield a scale dependence of the growth function, even at low redshifts, with a suppression at small scales. Leakage of gravity on a proper scale of  $\lambda \gtrsim 10 h^{-1}$  Mpc could nearly completely erase the signatures of non-zero neutrino mass on the shape of power spectrum, making the detection harder and softening the upper limit on the mass.

A modified cosmic expansion rate should not affect the scale-dependent distortion of the power spectrum shape due to Yukawa-like contributions. Whereas the exact dependence of the shape on both strength and cut-off length is slightly dependent on back-reaction from modified expanding background, a detection of deviation from the inverse-square law should be independent of the model for the cosmic expansion rate.

The degeneracies discussed suggest caution when determining limits on deviation from the inverse-square law from galaxy clustering using sharp priors. At the same time, constraints on cosmological parameters, such as  $\Omega_{M0}$  and  $f_v$ , could be affected by modified gravity physics. Full information from acoustic oscillations and evolution could disentangle some degeneracies. Furthermore, even if many imprints due to modified gravity are degenerate with other effects with respect to the shape, they are not with respect to normalization. While hot dark matter suppresses galaxy clustering on small scales, enhanced gravity boosts perturbation growth on large scales. At the same time, in a cosmic expansion enhanced by enhanced gravity, perturbations are strongly suppressed and the standard growth rate is achieved only on large scales.

We have discussed deviations from the inverse-square law in the standard framework of adiabatic initial conditions but it remains an open question whether a modified gravitational force could make room for isocurvature initial conditions (Peebles 2002).

Following a phenomenological approach, a Yukawa-like modification to the potential can be introduced without being derived from some specific gravity theory. Although such an empirical analysis can be enough to investigate the effects of deviations from the inverse-square gravity law, some further theoretical considerations can be of interest. Variation of the gravitational constant can arise naturally in scalar–tensor theories. Whereas a massive scalar field can produce a Yukawa-like modification (Zhytnikov & Nester 1994), the existence of a massless scalar field coupled to the tensor field of Einstein gravity can implement a time varying  $G$ . The Jordan–Brans–Dicke theory is the original and simplest extended theory of gravity which leads to time variations in  $G$  (Brans & Dicke 1961) and has provided a suitable framework for investigation of the effects of a changing  $G$  on structure formation (Liddle, Mazumdar & Barrow 1998; Nagata, Chiba & Sugiyama 2002). The main effect on the matter power spectrum is connected to a shift to a higher redshift of the matter–radiation equality which in turn leads to a shift of the maximum of the matter power spectrum and to more small-scale power.

A time-varying gravitational constant has an effect quite distinct from that of a spatially varying gravitational constant, as considered in this paper in the form of a Yukawa-like contribution to the potential. In fact, due to the lack of a characteristic scale during matter domination, a time-varying  $G$  will alter the growth rate but it will not change the shape of the spectrum, unlike a spatially varying  $G$ , which naturally introduces a cut-off length-scale. This main difference characterizes potential parameter degeneracies. As we have

seen, a Yukawa-like modification can act like hot dark matter; on the other hand, the shift in the power spectrum due to a time-varying  $G$  has a potential degeneracy with increasing the number of massless species. Effects of either spatial or temporal variations in  $G$  could be better probed via their effects on the overall normalization of the power spectrum and its evolution, although biasing is a major source of uncertainty.

Constraints on a Yukawa-like contribution to the potential have been found in the laboratory, and analyses of planetary motion in the Solar system can extend this to  $\lambda \sim 1$  au (Aldelberger et al. 2003). Such bounds on  $\Delta\alpha$  can be very tight, but they do not apply to scales beyond the Solar system and give no information for  $\lambda \gtrsim 10$  Mpc, i.e. the length-scales we have been considering with regard to structure formation. On these scales, cosmology offers the only probe of modified gravity models. At present, we have seen that the near-degeneracy between modified gravity and changes in the cosmological parameters means that exotic gravitational models remain hard to exclude. However, such degeneracies can be broken with good enough data (e.g. Sawicki & Carroll 2005) and we can look forward to increasingly stringent tests of Einstein gravity on the largest scales.

## ACKNOWLEDGMENTS

MS thanks the Institute for Astronomy, University of Edinburgh, for hospitality and financial support during his visit. JAP was supported by a PPARC Senior Research Fellowship. This work has been partially supported by the MIUR grant COFIN2004020323.

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